Dream Job Salary
Algebra 1 Performance Task

Instructions:

The following task contains 4 parts. We are interested in all of your mathematical thinking so please be sure to show your work and read the situation carefully. If at any time you get stuck feel free to try another part of the task. Relax. Be creative. Good Luck.

This is an imaginary story where mathematics is needed to understand the events.

The Situation: You are offered a 30-day trial period at your dream job. However, the owner of the company is a little unusual so the pay options are non-traditional. They offer you three different pay options for the 30 days.

Option 1: $60,000 a day

Option 2: You make one penny the first day, two pennies the second day, four pennies the third day, eight pennies the fourth day, etc.

Option 3: On your first day of work, you get $1. On your second day of work, you get $4. On your third day of work, you get $9. On your fourth day of work, you get $16. It continues this way for 30 days and then once you've completed the 30 days you receive a completion bonus of $500,000.

1.) Decide which payment option you would like to take (which earns you the most money?). In supporting your decision you must **use at least two representations** (tables, graphs, and/or equations) for each option and **explain** your decision. (Remember that this is about total pay after 30 days.)

A-CED.2
F-LE.1
F-LE.3
F-IF.4

Problem Solving / Strategizing
Reasoning & Proof
Communication

2.) Your unusual boss decided to offer you a fourth option to even complicate your decision. Your payment would be described by the function P(*x*) = 95,000x + 200,000 with *x* representing days you work and P representing dollars you earn. Explain the meaning of the function based on this situation and then decide if you would take this option over the other three choices. **Use a graph to support your decision.**

A-REI.3
F-LE.1
F-IF.4

Problem Solving / Strategizing
Reasoning & Proof
Communication

3) You decide to work until you make $1.5 million ($1,500,000) in the shortest amount of time. Using the four different plans you have been offered which option would you now chose? How does this affect the option you have chosen? Do you want to change your choice? Support your decision using at least two different representations. (Remember this is about total pay and you can use any of the four options.)

A-REI.3
F-LE.1
F-IF.4

Problem Solving / Strategizing
Reasoning & Proof
Communication

4) You have now thought about four functions. Please use your experience from your previous math classes and this task to compare these functions.

How are they similar? How are they different? Make sure you discuss the different ways they model growth, the different ways they can be represented and the key features of each function.

F-LE.1
F-LE.3
F-IF.4

Reasoning & Proof
Communication

This task will reveal students thinking related to practice standards MP1, MP2, MP3, MP6 & MP7 in part as described below.

Mathematical Thinking Dimensions	Common Core Mathematical Practice Standards
Problem Solving / Strategizing - Perseverance is hard to measure - CCS doesn't talk about perseverance in the description of MP1 at all, but rather the qualities of a proficient problem solver.	**MP1 –** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. **MP2 –** the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved….Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP6 –** They calculate accurately and efficiently **MP7 –** Mathematically proficient students look closely to discern a pattern or structure.
Reasoning and Proof - Reasoning is a larger notion than a viable argument. - In HS our central goal is to develop a broad notion of proof (see Prof. Wu) where argument is embedded within the idea of proof. - Critiquing the reasoning of others primarily takes place in the classroom and thus is not primarily something exhibited in a task.	**MP3 –** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusion…They reason inductively about data, making plausible arguments that take into account the context from which the data arose.
Communication - Communication is essential to all disciplines. Problem solving/strategizing, reasoning, representations, and mathematical connections are essential to mathematical communication. - We notice that the creators of the common core used the word communicate in different Practices and we felt it could be combined into one strand.	**MP1 –** Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. **MP3 –** communicate them (the justification of their conclusions) to others **MP6 –** They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem.

This task will reveal students understandings related to standards A-CED.2, A-REI.3, F-IF.4, F-LE.1, and F-LE.3

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.*

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Algebra 1: Dream Job Salary – Content Scoring Guide – Question 1

Question 1: A-CED.2; F-LE.1 F-LE.3; F-IF.4 Problem Solving / Strategizing Reasoning & Proof Communication	**0**	**1**	**2**	**3**	**4**
Question 1: **Decide which payment option you would like to take (which earns you the most money?)**. In supporting your decision you must **use at least two representations** (tables, graphs, and/or equations) for each option and **explain** your decision. (Remember that this is about total pay after 30 days.)	No attempt	An attempt is made to represent each option in one way. There can be errors in the representations.	The student represents the daily payments or the cumulative total payments for 2 of the 3 options correctly in one way. The student uses the representations to explain the decision. A student might have an error in not placing all the options in the same units (e.g. The student might leave the exponential option in pennies so his comparison might be flawed.) A student might make an error in generalizing (e.g. The student might generalize the equation to $y = 2^x$ rather than $y = 2^{x-1}$.) A student might make an error in graphing (e.g. The student might graph using an incorrect scale.)	The student represents the daily payments or the cumulative total payments for at least one of the options in two ways correctly and represents the other two in one way correctly. The student chooses the correct option and explains using the representations.	The student represents the daily payments or the cumulative total payments for each option correctly in two ways. The student chooses the correct option with an explanation using different representations. (e.g. A student shows that the exponential growth can be represented as $y = 2^{x-1}$ thus leading to the amount that is earned on Day 30. This is also reflected in the accompanying table or graph.) The student understands this is a summation of 30 days and is able to explain the choice without needing to sum up the totals (e.g. The linear result of $1,800,000 is the sum for 30 days which is less than the amount the person gets on Day 30 based on doubling the pennies. The 1, 4, 9 pattern will yield $900 on the 30th day. The sum of all the payments including the bonus would be less than the pennies paid on Day 30.) But a student can go beyond as represented below.*

*Going Beyond: This assessment can be taken to sophisticated places. For example in comparing the three total salary payments after one month the student represents each of the sums as equations. The sum of the exponential payment $.01+.02+.04+.08+.16…+ (.01)2^{29}$ can be written as $y = .01(2^x – 1)$ and the quadratic sum can be represented as $y=[x(x+1) (2x+1)]/6$. We would not expect students to be able to derive the equation for the sum of squares but some students might be able to derive the sum of the exponential payment.

Mathematical Thinking Rubric Question 1: Dream Job Salary Algebra 1

	0	*1*	*2*	*3*	*4*
PROBLEM SOLVING / STRATEGIZING	No strategy is chosen. Student lacked the problem solving strategies to enter into the task. Such as simplifying the problem or making a table.	A strategy is chosen that is flawed and will not lead to a solution. Student's work focuses totally on an arithmetic approach thus finds it difficult to discern a pattern or structure to lead to a more efficient approach (e.g. creates tables with flaws showing a lack of understanding of the patterns within quadratic and exponential growth.)	A partially correct strategy is chosen or a correct strategy for only solving part of the task is chosen. Student begins to generalize, but lacks the ability to abstract a given situation, represent it symbolically and manipulate the representations. (e.g. Student is not sure what to do with the generalization and their approach is primarily arithmetic.)	A correct strategy is chosen based on the mathematical situation in the task. Student is able to generalize and abstract a given situation, represent it symbolically and manipulate the representations, but there is a flaw in their process. (e.g. Student's approach can primarily be described as algebraic, but there was an error in the process.) Student didn't make adjustments in strategies thus creating errors within their process.	An efficient strategy is chosen and progresses toward a correct solution. Student efficiently abstracts a given situation, represents it symbolically and manipulates the representations as if they have a life of their own. If necessary, adjustments in strategy are made along the way, and/or alternative strategies are considered. (e.g. student discovers an error and reworks the approach) All parts of the solution must be correct.*
REASONING AND PROOF	The reasoning is unclear or non-existent.	An argument is begun with little or no evidence to support it. (e.g. Student is unable to reason with data, and thus if there was a conclusion is had little meaning.)	An argument is constructed with some evidence given. The conclusion is unclear based on the evidence. (e.g. Student struggled to reason with data, and though their argument was constructed it lacked justification, and struggled take into account the context.)	Construction of an argument that was not fully supported by the evidence given, leading to a conclusion. The reader needs to infer justification from the evidence given. (e.g. Student showed the ability to reason with data, but made plausible arguments that were not fully justified and/or struggled take into account the context.)	Construction of an argument based on evidence, leading to a logical conclusion where there is no need for inference by the reader. (e.g. Student showed the ability to reason with data, making plausible arguments that take into account the context.)
COMMUNICATION	Responses are unclear. Representations are inappropriate for the problem. Little or no communication of ideas is evident.	Responses are disorganized, but there is some communication of ideas. Representations do not accurately represent the problem. Familiar or everyday language is used in the explanation.	Responses convey some of their reasoning and process. Representations are present, but there is disconnect between various representations. Mostly familiar, everyday language is used in the explanation.	Responses are clear and sequenced in conveying their reasoning and process. Representations are present but not utilized in supporting the mathematical ideas in their argument or solution. Mathematical language and/or symbols are used throughout the solution to share and clarify ideas.	Responses are thorough and well constructed in conveying their reasoning and process. Uses multiple representations to support and illustrate the mathematical ideas in their argument or solution. Mathematical language and/or symbolic notation are used to consolidate mathematical thinking and to communicate ideas. It could be described as elegant. (e.g. Student is careful about units of measure, as well as setting up tables, graphs, and equations.)

*Note to assist in scoring: If a computational error is present it should only affect whether or not a student can receive a 4 in the Problem Solving/Strategizing dimension. In the other two dimensions a student can still earn a 4 if they exhibit the qualities described, even if the student is using an incorrect number. The reason for this is that an effective problem solver would revisit their solutions to see if they made sense or were accurate.

Algebra 1: Dream Job Salary – Content Scoring Guide – Question 2

Question 2: A-REI.3; F-LE.1 F-IF.4 Problem Solving / Strategizing Reasoning & Proof Communication	**0**	**1**	**2**	**3**
Question 2: Your unusual boss decided to offer you a fourth option to even complicate your decision. Your payment would be described by the function P(x) = 95,000x + 200,000 with x representing days you work. Explain the meaning of the function based on this situation and then decide if you would take this option over the other three choices. **Use a graph to support your decision.**	No attempt	The student is unable to explain the meaning of the function but was able to choose the correct option with explanation. (e.g. The student was able to state that the exponential sum would still be greater than the sum of the given function on day 30.)	The student is able to explain the meaning of the function in real world terms (e.g. This offer means you will earn $95,000 a day with a bonus of $200,000.) The student chooses the correct option with explanation using the function without an accompanying graph or uses an incorrect sketch of a graph. (e.g. The sum for the new option would be $3,050,000 while the pay on day 30 for the exponential is $5,368,709.12. The sum for the exponential is $10,737,418.23)	The student is able to explain the meaning of the function in real world terms (e.g. This offer means you will earn $95,000 with a bonus of $200,000.) The student chooses the correct option with explanation using an accurate sketch of graph to support the decision (e.g. An accurate graph would show that the exponential surpasses the fourth option between the 28th and 29th day.)

Mathematical Thinking Rubric Question 2: Dream Job Salary Algebra 1

	0	*1*	*2*	*3*	*4*
PROBLEM SOLVING / STRATEGIZING	No strategy is chosen. Student lacked the problem solving strategies to enter into the task. Such as simplifying the problem or making a table.	A strategy is chosen that is flawed and will not lead to a solution. Student's work focuses totally on an arithmetic approach thus finds it difficult to discern a pattern or structure to lead to a more efficient approach (e.g. creates tables with flaws showing a lack of understanding of the patterns within quadratic and exponential growth.)	A partially correct strategy is chosen or a correct strategy for only solving part of the task is chosen. Student begins to generalize, but lacks the ability to abstract a given situation, represent it symbolically and manipulate the representations. (e.g. Student is not sure what to do with the generalization and their approach is primarily arithmetic.)	A correct strategy is chosen based on the mathematical situation in the task. Student is able to generalize and abstract a given situation, represent it symbolically and manipulate the representations, but there is a flaw in their process. (e.g. Student's approach can primarily be described as algebraic, but there was an error in the process.) Student didn't make adjustments in strategies thus creating errors within their process.	An efficient strategy is chosen and progresses toward a correct solution. Student efficiently abstracts a given situation, represents it symbolically and manipulates the representations as if they have a life of their own. If necessary, adjustments in strategy are made along the way, and/or alternative strategies are considered. (e.g. student discovers an error and reworks the approach) All parts of the solution must be correct.*
REASONING AND PROOF	The reasoning is unclear or non-existent.	An argument is begun with little or no evidence to support it. (e.g. Student is unable to reason with data, and thus if there was a conclusion is had little meaning.)	An argument is constructed with some evidence given. The conclusion is unclear based on the evidence. (e.g. Student struggled to reason with data, and though their argument was constructed it lacked justification, and struggled take into account the context.)	Construction of an argument that was not fully supported by the evidence given, leading to a conclusion. The reader needs to infer justification from the evidence given. (e.g. Student showed the ability to reason with data, but made plausible arguments that were not fully justified and/or struggled take into account the context.)	Construction of an argument based on evidence, leading to a logical conclusion where there is no need for inference by the reader. (e.g. Student showed the ability to reason with data, making plausible arguments that take into account the context.)
COMMUNICATION	Responses are unclear. Representations are inappropriate for the problem. Little or no communication of ideas is evident.	Responses are disorganized, but there is some communication of ideas. Representations do not accurately represent the problem. Familiar or everyday language is used in the explanation.	Responses convey some of their reasoning and process. Representations are present, but there is disconnect between various representations. Mostly familiar, everyday language is used in the explanation.	Responses are clear and sequenced in conveying their reasoning and process. Representations are present but not utilized in supporting the mathematical ideas in their argument or solution. Mathematical language and/or symbols are used throughout the solution to share and clarify ideas.	Responses are thorough and well constructed in conveying their reasoning and process. Uses multiple representations to support and illustrate the mathematical ideas in their argument or solution. Mathematical language and/or symbolic notation are used to consolidate mathematical thinking and to communicate ideas. It could be described as elegant. (e.g. Student is careful about units of measure, as well as setting up tables, graphs, and equations.)

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Algebra 1: Dream Job Salary – Content Scoring Guide – Question 3

Question 3: A-REI.3; F-LE.1 F-IF.4 Problem Solving / Strategizing Reasoning & Proof Communication	**0**	**1**	**2**	**3**
Question 3: You decide to work until you make $1.5 million ($1,500,000) in the shortest amount of time. Using the four different plans you have been offered which option would you now chose? How does this affect the option you have chosen? Do you want to change your choice? Support your decision using at least two different representations. (Remember this is about total pay and you can use any of the four options.)	No attempt	The student was able to get to the correct solution without clear explanation. Or The student made an attempt to represent each option with errors.	The student was able to correctly explain the decision using one type of representation correctly. (e.g. A student compares each of the options using one representation and shows that the 4th option is the best. option.)	The student was able to correctly explain the decision using at least two different representations (e.g. A student compares the different options and shows with two representations that option 4 is the best option.)

Mathematical Thinking Rubric Question 3: Dream Job Salary Algebra 1

	0	*1*	*2*	*3*	*4*
PROBLEM SOLVING / STRATEGIZING	No strategy is chosen. Student lacked the problem solving strategies to enter into the task. Such as simplifying the problem or making a table.	A strategy is chosen that is flawed and will not lead to a solution. Student's work focuses totally on an arithmetic approach thus finds it difficult to discern a pattern or structure to lead to a more efficient approach (e.g. creates tables with flaws showing a lack of understanding of the patterns within quadratic and exponential growth.)	A partially correct strategy is chosen or a correct strategy for only solving part of the task is chosen. Student begins to generalize, but lacks the ability to abstract a given situation, represent it symbolically and manipulate the representations. (e.g. Student is not sure what to do with the generalization and their approach is primarily arithmetic.)	A correct strategy is chosen based on the mathematical situation in the task. Student is able to generalize and abstract a given situation, represent it symbolically and manipulate the representations, but there is a flaw in their process. (e.g. Student's approach can primarily be described as algebraic, but there was an error in the process.) Student didn't make adjustments in strategies thus creating errors within their process.	An efficient strategy is chosen and progresses toward a correct solution. Student efficiently abstracts a given situation, represents it symbolically and manipulates the representations as if they have a life of their own. If necessary, adjustments in strategy are made along the way, and/or alternative strategies are considered. (e.g. student discovers an error and reworks the approach) All parts of the solution must be correct.*
REASONING AND PROOF	The reasoning is unclear or non-existent.	An argument is begun with little or no evidence to support it. (e.g. Student is unable to reason with data, and thus if there was a conclusion is had little meaning.)	An argument is constructed with some evidence given. The conclusion is unclear based on the evidence. (e.g. Student struggled to reason with data, and though their argument was constructed it lacked justification, and struggled take into account the context.)	Construction of an argument that was not fully supported by the evidence given, leading to a conclusion. The reader needs to infer justification from the evidence given. (e.g. Student showed the ability to reason with data, but made plausible arguments that were not fully justified and/or struggled take into account the context.)	Construction of an argument based on evidence, leading to a logical conclusion where there is no need for inference by the reader. (e.g. Student showed the ability to reason with data, making plausible arguments that take into account the context.)
COMMUNICATION	Responses are unclear. Representations are inappropriate for the problem. Little or no communication of ideas is evident.	Responses are disorganized, but there is some communication of ideas. Representations do not accurately represent the problem. Familiar or everyday language is used in the explanation.	Responses convey some of their reasoning and process. Representations are present, but there is disconnect between various representations. Mostly familiar, everyday language is used in the explanation.	Responses are clear and sequenced in conveying their reasoning and process. Representations are present but not utilized in supporting the mathematical ideas in their argument or solution. Mathematical language and/or symbols are used throughout the solution to share and clarify ideas.	Responses are thorough and well constructed in conveying their reasoning and process. Uses multiple representations to support and illustrate the mathematical ideas in their argument or solution. Mathematical language and/or symbolic notation are used to consolidate mathematical thinking and to communicate ideas. It could be described as elegant. (e.g. Student is careful about units of measure, as well as setting up tables, graphs, and equations.)

*Note to assist in scoring: If a computational error is present it should only affect whether or not a student can receive a 4 in the Problem Solving/Strategizing dimension. In the other two dimensions a student can still earn a 4 if they exhibit the qualities described, even if the student is using an incorrect number. The reason for this is that an effective problem solver would revisit their solutions to see if they made sense or were accurate.

Algebra 1: Dream Job Salary – Content Scoring Guide – Question 4

Question 4: F-LE.1; F-LE.3 F-IF.4 Reasoning & Proof Communication	**0**	**1**	**2**	**3**	**4**
Question 4: You have now thought about four functions. Please use your experience from your previous math classes and this task to compare these functions. How are they similar? How are they different? Make sure you discuss the different ways they model growth, the different ways they can be represented and the key features of each function.	No attempt	The student was able to describe accurately the similarities and differences between linear, quadratic and exponential functions. He/she included in their discussion **at least one** of the following: rate of change, rate of change within context of the problem, y-intercepts, y-intercepts within context of the situations, end behavior, standard form of the different equations, meaning of the variables within each of the equations, comparing tables, comparing graphs, domain, range, domain and range within the context, and any other key feature of the three functions (e.g. presence of an asymptote in the exponential, or symmetry of the parabola)	The student was able to describe accurately the similarities and differences between linear, quadratic and exponential functions. He/she included in their discussion at **least two** of the following: rate of change, rate of change within context of the problem, y-intercepts, y-intercepts within context of the situations, end behavior, standard form of the different equations, meaning of the variables within each of the equations, comparing tables, comparing graphs, domain, range, domain and range within the context, and any other key feature of the three functions (e.g. presence of an asymptote in the exponential, or symmetry of the parabola)	The student was able to describe accurately the similarities and differences between linear, quadratic and exponential functions. He/she included in their discussion at **least three** of the following: rate of change, rate of change within context of the problem, y-intercepts, y-intercepts within context of the situations, end behavior, standard form of the different equations, meaning of the variables within each of the equations, comparing tables, comparing graphs, domain, range, domain and range within the context, and any other key feature of the three functions (e.g. presence of an asymptote in the exponential, or symmetry of the parabola)	The student was able to describe accurately the similarities and differences between linear, quadratic and exponential functions. He/she included in their discussion at **least four** of the following: rate of change, rate of change within context of the problem, y-intercepts, y-intercepts within context of the situations, end behavior, standard form of the different equations, meaning of the variables within each of the equations, comparing tables, comparing graphs, domain, range, domain and range within the context, and any other key feature of the three functions (e.g. presence of an asymptote in the exponential, or symmetry of the parabola)

Mathematical Thinking Rubric Question 4: Dream Job Salary Algebra 1

	0	*1*	*2*	*3*	*4*
REASONING AND PROOF	The reasoning is unclear or non-existent.	An argument is begun with little or no evidence to support it. (e.g. Student is unable to reason with data, and thus if there was a conclusion is had little meaning.)	An argument is constructed with some evidence given. The conclusion is unclear based on the evidence. (e.g. Student struggled to reason with data, and though their argument was constructed it lacked justification, and struggled take into account the context.)	Construction of an argument that was not fully supported by the evidence given, leading to a conclusion. The reader needs to infer justification from the evidence given. (e.g. Student showed the ability to reason with data, but made plausible arguments that were not fully justified and/or struggled take into account the context.)	Construction of an argument based on evidence, leading to a logical conclusion where there is no need for inference by the reader. (e.g. Student showed the ability to reason with data, making plausible arguments that take into account the context.)
COMMUNICATION	Responses are unclear. Representations are inappropriate for the problem. Little or no communication of ideas is evident.	Responses are disorganized, but there is some communication of ideas. Representations do not accurately represent the problem. Familiar or everyday language is used in the explanation.	Responses convey some of their reasoning and process. Representations are present, but there is disconnect between various representations. Mostly familiar, everyday language is used in the explanation.	Responses are clear and sequenced in conveying their reasoning and process. Representations are present but not utilized in supporting the mathematical ideas in their argument or solution. Mathematical language and/or symbols are used throughout the solution to share and clarify ideas.	Responses are thorough and well constructed in conveying their reasoning and process. Uses multiple representations to support and illustrate the mathematical ideas in their argument or solution. Mathematical language and/or symbolic notation are used to consolidate mathematical thinking and to communicate ideas. It could be described as elegant. (e.g. Student is careful about units of measure, as well as setting up tables, graphs, and equations.)